Spin Gap and Superconductivity in the Interlayer Pair Tunneling Model

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Abstract

A simple interlayer pair tunneling model is solved exactly. We find that in the normal state the spin-$\frac{1}{2}$ particle and hole excitations are gapped. But the state is an unusual metal, characterized by novel fermionic, spin-zero and charge $\pm 2e$ gapless excitations that exist about new type of Fermi surfaces. The model is consistent with a number of unusual properties of underdoped cuprates. Superconductivity is induced by an additional intra-layer pairing interaction which opens a gap in the charge spectrum. The symmetry of the order parameter is in general different from the symmetry of the single-electron (or hole) gap. The former can be a d-wave, while the the latter is more complex.

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In underdoped bilayer cuprate superconductors there appears to be a gap, known as the spin gap, in the normal-state spectrum. For example, below about 150 K, magnetic susceptibility χ and NMR relaxation rate decrease rapidly [1]. Yet there is apparently no charge gap since the in-plane conductivity is metallic. Recent photoemission experiments further show that only a fraction of the the Fermi surface near $(\pi, 0)$ and symmetric points seems to be gapped [2]. On continuity grounds, Fermi or Luttinger liquids are not expected to show such a behavior, suggesting that these systems may belong to a different universality class. Since the spin-gap has been observed only in bilayer systems, it has been suggested that interlayer interactions may be responsible [3]. An interesting possibility is interlayer pair tunneling which was initially considered as a mechanism for T_c enhancement [4]. The underlying assumption is that single electrons can not hop coherently between copper-oxide layers so that second order hopping processes such as pair tunneling become important.

Recently Chakravarty *et al.* have studied a simple model of pair tunneling between two layers and found considerable enhancement of T_c within a mean-field approximation [5]. Physics within each layer is described by a Bardeen, Cooper and Schrieffer (BCS) type reduced Hamiltonian, and tunneling is also confined to the reduced subspace of pairs with zero total momentum. The mean-field state does not have a spin gap. But, as shown by Anderson, when the tunneling interaction is treated exactly the normal-state χ exhibits spin-gap behavior [6]. In this paper we show that, although the model is too simple to provide a detailed description of the cuprates, it has a number interesting properties. We find that in the normal state spin-$\frac{1}{2}$ electron and hole excitations acquire a gap. The Fermi surface appears partially gapped at finite T (experimental situation). An important property is that there are gapless spin-0, charge $\pm 2e$ excitations which are *fermionic* and have their own Fermi surface, and which dominate the low-T physics. The system is thus metallic, but not a Fermi liquid in the usual sense. To induce superconductivity we add an intralayer attractive interaction which opens a gap in the charge spectrum. Therefore the symmetry of the charge gap (and hence the order parameter) is determined by the attractive interaction and is in general different from the symmetry of the one particle (i.e., spin-) gap.

We consider the following Hamiltonian for two layers

$$H = \sum_{i\mathbf{k}\sigma} \epsilon(\mathbf{k}) c_{i,\mathbf{k}\sigma}^\dagger c_{i\mathbf{k}\sigma} - \sum_{\mathbf{k},\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} c_{i\mathbf{k}\uparrow}^\dagger c_{i,-\mathbf{k}\downarrow}^\dagger c_{i,-\mathbf{k}'\downarrow} c_{i,\mathbf{k}'\uparrow} - \sum_{\mathbf{k}} T_J(\mathbf{k})[c_{1,\mathbf{k}\uparrow}^\dagger c_{1,-\mathbf{k}\downarrow}^\dagger c_{2,-\mathbf{k}\downarrow} c_{2,\mathbf{k}\uparrow} + h.c.].$$

(1)

Here $c_{i,\mathbf{k}\sigma}$ destroys an electron carrying a (two-dimensional) wavevector $\mathbf{k}$, spin $\sigma = \uparrow, \downarrow$ in layer $i = 1, 2$. The first term describes an electron band of energy $\epsilon(\mathbf{k})$ for each layer. The second term is an in-plane attractive interaction and the third term decribes tunneling. Like

the BCS Hamiltonian only pairs of zero total momentum are considered in the interaction terms so that tunneling matrix element $T_J(\mathbf{k})$ is diagonal in $\mathbf{k}$. The assumption is that $T_J(\mathbf{k})$ is O(1). This is different from more traditional interactions such as $V_{\mathbf{k}\mathbf{k}'}$ (or tunneling terms with nonzero momentum) which scale inversely as the volume of the system and hence does not contribute for $\mathbf{k} = \mathbf{k}'$. It is precisely the diagonal nature of $T_J(\mathbf{k})$ that causes the enhancement of T_c and the spin gap. Although the problem can be solved for any $T_J(\mathbf{k})$, for definiteness we will take

$$T_J(\mathbf{k}) = \frac{t_J}{16}(\cos k_x - \cos k_y)^4. \tag{2}$$

This is the same form used in the mean-field analysis [5]. Our focus will be on the universal features and as such we will take a simple cosine band: $\epsilon(\mathbf{k}) = -2t[\cos k_x a + \cos k_y a]$.

We first ignore V and solve the normal state problem. Then the Hamiltonian can be diagonalized exactly by diagonalizing each $\mathbf{k}$ subspace separately [7]. For a given $\mathbf{k}$, we need to consider four single-electron states: $(1, \mathbf{k} \uparrow), (1, -\mathbf{k} \downarrow), (2, \mathbf{k} \uparrow)$ and $(2, -\mathbf{k} \downarrow)$. It is convenient to diagonalize $\mathcal{H}_{\mathbf{k}} = H_{\mathbf{k}} - \mu N_{\mathbf{k}}$, where $N_{\mathbf{k}}$ is the number operator and μ is the chemical potential. Let $\xi_{\mathbf{k}} = \epsilon(\mathbf{k}) - \mu$. There are 16 many-body states which can be grouped according to the total electron number $N_{\mathbf{k}}$ and z-component of total spin S_z which are both conserved. In the absence of tunneling, an eigenstate has energy $N_{\mathbf{k}}\xi_{\mathbf{k}}$, with $N_{\mathbf{k}} = 0, 1, 2, 3, 4$. We represent such a state by $|a \quad b>$, where a (b) stands for one of the four states in the first (second) layer: $0, \uparrow, \downarrow, \uparrow\downarrow$. Tunneling only connects two of these: $|\uparrow\downarrow \quad 0>$ and $|0 \quad \uparrow\downarrow>$, with $N_{\mathbf{k}} = 2$ and $S_z = 0$. Diagonalizing the 2 by 2 matrix leads to the following two states:

$$|2\pm> = \frac{1}{\sqrt{2}}[|\uparrow\downarrow \quad 0> \pm |0 \quad \uparrow\downarrow>], \tag{3}$$

with energies $2\xi_{\mathbf{k}} \mp T_J(\mathbf{k})$, respectively. The remaining fourteen states are unaffected. Note that since $N_{\mathbf{k}}$ is conserved, there is no long-range order, i.e, there is no superconductivity in the absence of V.

Ground State: The ground state of the full Hamiltonian is constructed by selecting the lowest energy state of $\mathcal{H}_{\mathbf{k}}$ for each $\mathbf{k}$. The relevant states are the (i) the four-electron state $|4> \equiv |\uparrow\downarrow \quad \uparrow\downarrow>$ of energy $4\xi_{\mathbf{k}}$; (ii) the zero-electron state $|0> \equiv |0 \quad 0>$ of energy 0 and (iii) the two-electron state $|2+>$ of energy $2\xi_{\mathbf{k}} - T_J(\mathbf{k})$. For $T_J = 0$, states with different $N_{\mathbf{k}}$ cross at the Fermi level which is at $\xi_{\mathbf{k}} = 0$. So one must pick the four-electron state for $\xi_{\mathbf{k}} < 0$ and the zero-electron state for $\xi_{\mathbf{k}} > 0$.

For nonzero $T_J(\mathbf{k})$, the four-electron state still has the lowest energy for $\xi_{\mathbf{k}} < -\frac{1}{2}T_J(\mathbf{k})$ and the zero-electron state for $\xi_{\mathbf{k}} > \frac{1}{2}T_J(\mathbf{k})$. But in the region $-\frac{1}{2}T_J(\mathbf{k}) < \xi_{\mathbf{k}} < \frac{1}{2}T_J(\mathbf{k})$,

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the two-electron state $|2+>$ has the lowest energy. We will call this the two-electron region. *Therefore the original Fermi surface has disappeared, except at points where the line* $T_J(\mathbf{k}) = 0$ *intersects the Fermi surface ($\xi_\mathbf{k} = 0$).* In its place, two new "Fermi" surfaces have appeared. Surface I is at $\xi_\mathbf{k} = -\frac{1}{2}T_J(\mathbf{k})$ and separates the four-electron states from the two-electron states. Surface II appears at $\xi_\mathbf{k} = \frac{1}{2}T_J(\mathbf{k})$ and separates the two-electron states from the zero-electron states. As shown in Fig 1, the particle number $N_\mathbf{k}$ has discontinuities at the new Fermi surfaces whose topolgy depends on the form of $T_J(\mathbf{k})$. In the quartic case, $T_J(\mathbf{k})$ is largest near $(0, \pm\pi)$ and $(\pm\pi, 0)$, but it vanishes along $k_x = k_y$, so that the original Fermi surface survives at four points.

Excitations: In the absence of tunneling, the elementary excitations are the usual holes, created by picking a three-electron state in the four-electron region and particles, created by picking a one-electron state in the zero-electron region. These are gapless (energy $|\xi_\mathbf{k}|$), carry charge $\pm e$ and spin $\frac{1}{2}$.

(i) For $T_J(\mathbf{k}) > 0$, the hole and particle excitations described above exist for $\xi_\mathbf{k} < -\frac{1}{2}T_J(\mathbf{k})$ and $\xi_\mathbf{k} > \frac{1}{2}T_J(\mathbf{k})$, respectively. Two additional spin-$\frac{1}{2}$ excitations exist in the two-electron region: a particle of energy $\xi_\mathbf{k} + T_J(\mathbf{k})$ and a hole of energy $-\xi_\mathbf{k} + T_J(\mathbf{k})$. *All four spin-$\frac{1}{2}$ excitations are gapped.* Therefore, magnetic susceptibility χ is zero at $T = 0$. The "spin gap" is thus a natural consequence of tunneling, will be observed in photoemission experiments. We define a gap function $\Delta_{spin}(\mathbf{k})$ by the value of the excitation energy at the new Fermi surface. This equals $\frac{1}{2}T_J(\mathbf{k})$, exactly the distance between the old and new Fermi surfaces.

(ii) Near the new Fermi surfaces there are additional *gapless* hole- and particle-type excitations. However, they have spin zero and charge $\pm 2e$. Near surface I, a particle is created by selecting the four-electron state in the two-electron region and a hole is created by selecting a two-electron state in the four-electron region. These have energy $\pm(2\xi_\mathbf{k} + T_J(\mathbf{k}))$ which vanish at surface I ($\xi_\mathbf{k} = -\frac{1}{2}T_J(\mathbf{k})$), and can not be created from two spin-$\frac{1}{2}$ excitations. Similar spin-zero, charge-two particle and hole excitations also exist near Fermi surface II with energies $\pm(2\xi_\mathbf{k} - T_J(\mathbf{k}))$.

In short, while there is a spin gap, there is no *charge gap*. The charge excitations can not be viewed as 'preformed' Cooper pairs since, as shown below, they behave like fermions, at least as far as thermodynamic properties are concerned. The partition function for the $\mathbf{k}$ subspace is given by

$$Z(\mathbf{k}) = Z_0(\mathbf{k}) + Z_1(\mathbf{k}) = (1 + e^{-\beta\xi_\mathbf{k}})^4 + 2[\cosh(\beta T_J(\mathbf{k})) - 1]e^{-2\beta\xi_\mathbf{k}}, \qquad (4)$$

where Z_0 is the partition function for $T_J = 0$, and $\beta = 1/kT$. Thermodynamic properties are largely determined by the behavior of the two factors Z_0/Z and Z_1/Z as a function of

T. At high temperatures, $kT >> T_J$, $Z_1 \to 0$ and we recover the noninteracting result. But at low temperatures such that $\beta T_J >> 1$ and $\beta |\xi| >> 1$, the two factors look like Fermi functions. Thus for $\xi < 0$

$$Z_0/Z \sim f(2\xi + T_J), \quad Z_1/Z \sim (1 - f(2\xi + T_J)). \tag{5}$$

And for $\xi > 0$

$$Z_0/Z \sim (1 - f(2\xi - T_J)), \quad Z_1/Z \sim f(2\xi - T_J). \tag{6}$$

Note that these functions have step discontinuities at the new Fermi surfaces, and the energy argument correspond to the charge excitations. Of course, far from the Fermi surfaces they lose their fermionic character.

We will be interested in the case for which the noninteracting Fermi energy $\epsilon_{F0} >> t_J, kT$. As shown in Fig. 1, for $kT << T_J$, the new Fermi surfaces and the fermionic character of the charge excitations are evident in the particle number $n_{\mathbf{k}} \equiv N_{\mathbf{k}}/4$, which is given by

$$n_{\mathbf{k}} = \frac{Z_0(\mathbf{k})}{Z(\mathbf{k})} f(\xi_{\mathbf{k}}) + \frac{Z_1(\mathbf{k})}{2Z(\mathbf{k})}. \tag{7}$$

With increasing T, the charge excitations break up and density of spin-$\frac{1}{2}$ excitations increases, and at $kT \sim T_J$, the new Fermi surfaces have effectively disappeared so that $n_{\mathbf{k}}$ looks like the Fermi distribution function for the noninteracting system. Therefore the gap $\Delta_{spin}(\mathbf{k})$ will be seen only on those parts of the Fermi surface where $T_J(\mathbf{k})$ is large compared to kT. For the quartic case, $T_J(\mathbf{k})$ vanishes at four points along the (1,1) direction, and is quite small (because of the fourth power) except in the neighborhood of $(\pm\pi, 0)$ and $(0, \pm\pi)$, where the gap is largest. Hence, for $T > 0$, parts of the Fermi surface along (1,1) and nearby directions will appear to be ungapped. For a fixed T, the gap decreases with doping, as the old Fermi surface moves away from the maximal points. For fixed density it effectively disappears for $kT > t_J$. These results are in qualitative agreement with recent photoemission measurements [2].

The state is clearly not the usual Fermi liquid. This is true for arbitrarily small t_J, and all densities and spatial dimensions (there is no quantum phase transition). Moreover, there is no broken symmetry. For $kT << t_J$, charge fermions dominate and make the system a degenerate metal. With increasing T these give way to spin-$\frac{1}{2}$ excitations, and eventually at $kT \sim t_J$ there is a crossover to an ordinary Fermi liquid.

The susceptibility is determined by the spin-$\frac{1}{2}$ excitations, and is given by [6]

$$\chi = \frac{\mu_0^2}{2N} \sum_{\mathbf{k}} \frac{Z_0(\mathbf{k})}{Z(\mathbf{k})} \left[\frac{\beta e^{\beta \xi_{\mathbf{k}}}}{(e^{\beta \xi_{\mathbf{k}}} + 1)^2} \right], \tag{8}$$

where $\mu_0 = e\hbar/2mc$. The quantity in the bracket is localized at the old Fermi surface ($\xi = 0$) and has width kT. For $kT < T_J$, its overlap with the factor Z_0/Z is exponentially small. Thus if $T_J(\mathbf{k})$ has no zeroes at the Fermi level (finite gap), then χ vanishes exponentially with T. Suppose $T_J(\mathbf{k})$ is a constant $= 2\Delta_{spin}$. Then, for $kT << T_J << \epsilon_{F0}$, we find that $\chi/\chi_0 \approx (kT/\Delta_{spin})exp(-2\Delta_{spin}/kT)$, where $\chi_0 = \frac{1}{2}\rho_0\mu_0^2$ is the zero-temperature susceptibility for the noninteracting system and ρ_0 is the corresponding density of states at the Fermi level. For $kT >> t_J$, on the other hand, we recover the noninteracting result: $\chi/\chi_0 \approx 1 - \frac{2}{15}(\Delta_{spin}/kT)^2$.

If the gap function has zeroes one expects a power law. The power depends on the symmetry of $\Delta_{spin}(\mathbf{k})$. For the quartic case, $T_J(\mathbf{k})$ (and hence the gap) vanishes at the Fermi points $\mathbf{k}_0$ as $T_J(\mathbf{k}) \propto (q_x - q_y)^4$, where $\mathbf{q} = \mathbf{k} - \mathbf{k}_0$. Then we find that χ vanishes as $\chi \sim T^{1/4}$. More generally $\chi \sim T^{1/p}$, if the spin gap vanishes with a power p. Fig 2. shows the temperature dependence of χ, calculated numerically, for several values of the electron density n. We see that χ deviates sharply from χ_0 below a temperature T_{sus} and then decreases slowly as $T^{1/4}$. The scale T_{sus} decreases with decreasing n.

For $T \to 0$ charge-fermions dominate. For example, specific heat per site $C(T) \equiv T\gamma(T)$ vanishes linearly with T. Fig. 3 shows the temperature dependence of $\gamma(T)/\gamma_0$, where γ_0 is the coefficient for the noninteracting system. We find that at low T, $\gamma(T)/\gamma_0 \approx 1/4 + aT^{1/4}$. The constant term $1/4$ arises because, for charge fermions, the spin-degeneracy is reduced by a factor of 2 and their Fermi energy is incresed by a factor of 2, to leading order in t_J/ϵ_{F0}. The $T^{1/4}$ is again the contribution from the spin-$\frac{1}{2}$ excitations. As shown in Fig. 3, $\gamma(T)$ goes through broad maximum at $T = T_{spht}$, and at large T approaches the nonointeracting value γ_0. Thus the Wilson ratio ($\propto C/T\chi$) is temperature dependent and becomes infinite at $T = 0$. The scale T_{spht} is in general much smaller than T_{sus}.

Superconductivity: To induce superconductivity we now add the in-plane interaction $V_{\mathbf{k},\mathbf{k'}}$. It is sufficient to treat this term by mean-field approximation since it is restricted to the reduced subspace of zero-momentum pairs. But T_J is again treated exactly. Let $b_{\mathbf{k}} \equiv <c_{i\mathbf{k}\uparrow}^\dagger c_{i,-\mathbf{k}\downarrow}^\dagger>$ be the order-parameter which we take to be real and independent of the layer index. Then the mean-field pairing Hamiltonian is

$$H_{super,MF} = -\sum_{i,\mathbf{k}} \Delta_{\mathbf{k}} \left[c_{i,\mathbf{k}\uparrow}^\dagger c_{i,-\mathbf{k}\downarrow}^\dagger + c_{i,-\mathbf{k}\downarrow} c_{i,\mathbf{k}\uparrow} \right], \tag{9}$$

where $\Delta_{\mathbf{k}} = \sum_{\mathbf{k'}} V_{\mathbf{k}\mathbf{k'}} b_{\mathbf{k'}}$. The full Hamiltonian can again be diagonalized for each $\mathbf{k}$ separately. The total S_z is still a good quantum number for each $\mathbf{k}$, but $N_{\mathbf{k}}$ is no longer conserved. States with $S_z \neq 0$ are not affected by tunneling, and therefore are the same as in the BCS theory. In particular, the $S_z = \pm 1/2$ subspaces split into two manifolds of energy $2\xi_{\mathbf{k}} \pm R_{\mathbf{k}}$,

where $R_{\mathbf{k}} = (\xi_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2)^{\frac{1}{2}}$.

The lowest-energy eigenstate belongs to the same $S_z = 0$ subspace as before, the subspace consisting of the three states $|0>$, $|2+>$ and $|4>$. These are now coupled by the pairing Hamiltonian leading to the following 3 by 3 matrix:

$$\begin{vmatrix} 0 & -\sqrt{2}\Delta_{\mathbf{k}} & 0 \\ -\sqrt{2}\Delta_{\mathbf{k}} & 2\xi_{\mathbf{k}} - T_J(\mathbf{k}) & -\sqrt{2}\Delta_{\mathbf{k}} \\ 0 & -\sqrt{2}\Delta_{\mathbf{k}} & 4\xi_{\mathbf{k}} \end{vmatrix}.$$

The eigenvalues are given by

$$\Lambda_m = 2\xi_{\mathbf{k}} - \frac{1}{3}T_J(\mathbf{k}) + 2\left(\frac{d}{3}\right)^{\frac{1}{2}}\cos[\frac{1}{3}(2\pi m + \phi)], \tag{10}$$

where $m = 1, 2, 3$, and $\tan\phi = -\left(\frac{4d^3}{27} - c^2\right)^{\frac{1}{2}}/c$, $d = 4\xi_{\mathbf{k}}^2 + 4\Delta_{\mathbf{k}}^2 + \frac{1}{3}T_J^2(\mathbf{k})$ and $c = \frac{2}{3}T_J(\mathbf{k})\left[2\Delta_{\mathbf{k}}^2 + \frac{1}{9}T_J^2(\mathbf{k}) - 4\xi_{\mathbf{k}}^2\right]$.

a. <u>Charge Gap:</u> For $T_J = 0$ we have the usual BCS problem. The spin-$\frac{1}{2}$ particles and holes acquire a gap $\Delta_{\mathbf{k}}$ at the original Fermi surface. For $T_J \neq 0$, these are already gapped in the normal state. The main source of the condensation energy is a gap that opens up in the charge fermion spectrum. For example, $|0>$ and $|2+>$ are degenerate with energy 0. at the (new) right Fermi surface ($\xi = T_J/2$). This is split by pairing, giving a charge gap which has the form $\Delta_{charge}(\mathbf{k}) = \Delta_{\mathbf{k}} f_1(\frac{\Delta(\mathbf{k})}{2T_J(\mathbf{k})})$, where $f_1(x)$ is even in x. For $\Delta_{\mathbf{k}}$ small compared to $2T_J(\mathbf{k})$, we find that $\Delta_{charge}(\mathbf{k}) \approx 2\sqrt{2}\Delta_{\mathbf{k}}$. The superconducting T_c is enhanced relative to the BCS value in part because $\Delta_{charge}(\mathbf{k})$ is generally larger than $\Delta_{\mathbf{k}}$.

b. <u>Symmetry of the Order Parameter:</u> The order parameter $b_{\mathbf{k}} = < c_{1\mathbf{k}\uparrow}c_{1-\mathbf{k}\downarrow} >$ is an *odd* function of $\Delta_{\mathbf{k}}$ since a change of sign $\Delta_{\mathbf{k}} \to -\Delta_{\mathbf{k}}$ is undone by a gauge transformation $c_{\mathbf{k}\uparrow} \to -c_{\mathbf{k}\uparrow}$ which changes the sign of $b_{\mathbf{k}}$. At $T = 0$ it has the form

$$b_{\mathbf{k}} = \Delta_{\mathbf{k}}\left[\frac{(4\xi - \Lambda(\mathbf{k}))^{-1} - \Lambda^{-1}(\mathbf{k})}{1 + 2\Delta_{\mathbf{k}}^2\left(\Lambda(\mathbf{k})^{-2} + (4\xi - \Lambda(\mathbf{k}))^{-2}\right)}\right], \tag{11}$$

where $\Lambda(\mathbf{k})$ is the lowest eigenvalue of the 3 by 3 matrix and is even in $\Delta_{\mathbf{k}}$. The symmetry of the order parameter (and of the charge gap) is thus determined by the symmetry of the in-plane gap function $\Delta_{\mathbf{k}}$, which is determined by our choice of $V_{\mathbf{k},\mathbf{k}'}$. In particular it could have $d_{x^2-y^2}$ symmetry. On the other hand, the gap in the spin-$\frac{1}{2}$ electron and hole excitations — the analog of the BCS gap — does not have the same symmetry since it now depends both on $T_J(\mathbf{k})$ and $\Delta_{\mathbf{k}}$. In other words, experiments that probe the order parameter (e.g., Josephson

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tunneling) could find a d-wave symmetry, but experiments in which single electrons are added or removed (e.g., photoemission) would see a more complex gap structure.

A number of other results can be obtained independently of the origin or details of $V_{\mathbf{k},\mathbf{k'}}$. As in the BCS model, $V_{\mathbf{k},\mathbf{k'}}$ is presumed to be appreciable near the original Fermi surface (at $\xi_{\mathbf{k}} = 0$), i.e., for $\xi_{\mathbf{k}}, \xi_{\mathbf{k'}} < \omega_0$, where ω_0 is the analog of the Debye energy. Consequently $\Delta_{\mathbf{k}}$ decreases with distance from $\xi_{\mathbf{k}} = 0$. But the charge gap scales with the value of $\Delta_{\mathbf{k}}$ at the new Fermi surface which is at a distance $\frac{1}{2}T_J(\mathbf{k}) = \Delta_{spin}(\mathbf{k})$. It follows that $\Delta_{charge}(\mathbf{k})$ is small where $\Delta_{spin}(\mathbf{k})$ is large (e.g., near $\pm\pi, 0$). This implies that close to half filling (underdoped regime) charge gaps are small, and increases with doping (decreasing n). Therefore, in this region, (1) T_c also increases with doping, as observed. (2) For fixed $T << T_c$, density of charge excitations *decreases* with doping. Consquently, electronic contribution to specific heat, thermal conductivity etc would decrease with increasing doping. (3) Since T_c increases and T_{sus} decreases with doping, the temperature range over which the spin gap is seen decreases with increasing doping and eventually disappears.

In conclusion, the interlayer pair tunneling model provides a simple example of the break down of Landau theory and can account for a number of unusual properties of the underdoped cuprates. Since the effect is caused by a $\mathbf{k}$-diagonal tunneling interaction one may ask whether such a restriction is physically justified. For example, a more general tunneling Hamiltonian which results from second order hopping processes would have the form

$$H_J = \sum_{\mathbf{k},\mathbf{q}} T_J(\mathbf{k},\mathbf{q})[c_{1,\mathbf{k}\uparrow}^{\dagger} c_{1,-\mathbf{k}+\mathbf{q}\downarrow}^{\dagger} c_{2,-\mathbf{k}+\mathbf{q}\downarrow} c_{2,\mathbf{k}\uparrow} \quad + \quad h.c.], \tag{12}$$

where $\mathbf{q}$ is the total momentum of the pair. Now, extensivity of free energy requires that $\sum_{\mathbf{k},\mathbf{q}} T_J(\mathbf{k},\mathbf{q})$ scales with the volume V, so that $T_J(\mathbf{k}\mathbf{q}) \propto V^{-1}$. But Hamiltonian (1) is obitained if only $\mathbf{q} = 0$ terms are kept. This is not thermodynamically significant unless one postulates that $T_J(\mathbf{k},\mathbf{q} = 0)$ is O(1), whereas terms with $\mathbf{q} \neq 0$ scale as V^{-1} and hence are similar to other interaction processes. Note that this implies the existence of something akin to a condensate which favors zero-momentum pair tunneling. Phenomenologically, if the diagonal terms are responsible for the spin-gap state, then thermodynamic considerations alone guarantee that they are fundamentally different from other interaction processes. It then makes sense to start with diagonal terms only, and treat the remainder as fluctuatuations. Generalization to many layers is, in principle, straightforward. Indeed Baskaran has shown that, if V is ignored, the model can be mapped onto a solvable quantum spin-chain model along the c-axis for each $\mathbf{k}$, provided the Hamiltonian is restricted to the $N_{\mathbf{k}} = even$ (i.e., charge) subspace only [9]. Then it is not clear what happens to the spin gap. Unfortunately, when V is included the spin-chain model is no longer solvable.

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Figure Captions

Fig. 1. Particle density $N_{\mathbf{k}}$ along the $(\pi, 0)$ direction. The intermediate step with $N_{vk} = 2$ is the two-electron region. At low $T = 0.1t_J$ ($k_B = 1$) there are two Fermi functions representing charge $\pm 2e$ excitations. But at $T = 0.5t_J$, there is only one Function corresponding to the nontinteracting FS (which is where the three lines intersect). Bandwidth $= 40t_J$.

Fig 2. (a) Magnetic susceptibility χ/χ_0 as a function of scaled temperature t/t_J, for four values of electron density n. Note that characteristic temperature decreases with decreaing

n. (b) At low T, χ vanishes as $T^{1/4}$, as shown in the log-log plot. The slope of the line is $1/4$.

Fig 3. (a) Scaled specific heat cefficient $\gamma_{(}T)/\gamma_0 = C(T)/C_0(T)$ as a function of of scaled temperature T/t_J. The broad maximum appears at a temperature which is much lower than temperature at which susceptibility drops (see Fig 2a) and may not be seen if superconductivity intervenes. (b) The same quantity on a log-log plate showing that it approaches a constant ($=1/4$ + corrections) as $T \to 0$.